Numerical Solution of Diffusions Effects on Magnetohydrodynamic (MHD) Stagnation Point Nanofluid Flow with Chemical Reaction

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Abstract: Heat, mass, and flow dynamics transport within magnetohydrodynamic (MHD) hybrid nanofluids are thoroughly examined in this work. The joint impact of Dufour alongside Soret diffusions over Magnetohydrodynamic (MHD) stationary phase flow under the influence of chemical reaction is examined across a shrinking surface with partial slip. The solution is examined and analyzed using temperature, velocity, in addition to nanoparticle concentration. Following the application of suitable similarity solutions, the modified system of stream equations was then encountered utilizing the Shooting Technique in conjunction with the 4th-order Runge-Kutta technique. Numerical solutions' visual representations are investigated. The results demonstrated that higher magnetic parameter values result in a drop in the momentum profile. On the other hand, the system's temperature is raised by the velocity slip, Dufour diffusivity, chemical reaction, and radiation parameters. However, when Prandtl number and Soret diffusivity are influenced, the system's temperature is raised. On the other hand, the concentration profile is improved by raising the Prandtl number alongside radiation parameter. Conversely, the Nusselt number profile is diminished by the Prandtl number. The current approach has various practical applications in glass fiber production, polymer technology, and metallurgical processes such as annealing and tinning copper wires and chilling continuous strips or filaments by pressing through quiescent fluids.

Keywords: MHD, Soret diffusivity, Dufour diffusivity, Chemical reaction, Stagnation point, partial slip

1. Introduction

Magnetohydrodynamics (MHD) flow is the investigation of fluids exposed to magnetic fields. Currents are created by this applied force, and the Lorentz force, a retarding force, affects the flow. The final product's quality in heat exchange processes is greatly influenced by the cooling rate. Therefore, in order to regulate the cooling rate and provide the necessary high-quality goods, an MHD parameter needs to be taken into account.

Power generators, heat exchangers, MHD pumps, magnetic drug targeting, and the petroleum industry are just a few of the fluid flow scenarios in which MHD is employed [1] − [4]. Investigation of the buoyancy influence in the standstill point flow of a radiative bioconvective MHD nanofluid using Darcy Forchheimer porous media was conducted by [5]. Marangoni convection in stagnation point flow, a non-Newtonian flow through an unsteady stretched sheet, was empirically studied by [6]. A research investigation conducted of a non-Newtonian flow across a porous stretched sheet was given by Kumar [7]. And [8] modeled activation energy effects in Casson fluid, incorporating microorganisms over a disk. A similarity investigation into MHD non-Newtonian flow with a heat source on a porous extended sheet was carried out by [9].

While [10] investigated the radiative flow of MHD non-Newtonian across an exponential stretched sheet with viscous dissipation. The velocity of the flow field is zero with respect to the body and the particles in the fluid at the site of stagnation. The Bernoulli equation states that when the velocity is at rest, static pressure reaches its maximum value. Among the first researchers to introduce the idea of 2-D stagnation point flows were [11]. Glass, metallic and plastic sheets, metal spinning, hot rolling, and many other processes heavily rely on heat transfer and boundary layer flow caused by mobile surfaces. Glass blowing, fiber spinning, and continuous metal casting all exhibit flow characteristics on a stretch sheet. MHD blended convection stagnation point flow on a vertically extended sheet with a heat source/sink was investigated by [12].

A simple and incredibly stable system called the axisymmetric stationary point flow is produced when two identical segments maintain skin contact with mass and heat exchange near the stagnation area and flow rate. Furthermore, thermal oil recovery, transpiration cooling, reducing drag at the corner's edge, and the employment of push bearings and spiral diffusers are some of the basic applications for stagnation places. The first research of stagnation point flow induced by a stretched surface that extends over a plate was conducted by [13].

Mahabaleshwar et. al. [14] examined heat transfer over a decreasing sheet and the equilibrium of MHD stagnation point flowing in a viscous dissipation condition. Hussaini et. al. [15] investigated the magnetohydrodynamic transmission of a non-Newtonian fluid over an exponentially increasing surface.The heat transmission along with chemical reaction of MHD flow with Soret-Dufour phenomenon across an exponentially extending sheet were numerically studied by [16]. The results of fluid flow on a stretching surface may not be unique in the stagnation point flow. Similarly, there are frequently several methods for Newtonian fluids, in contrast to non-Newtonian fluids. It is crucial to employ partial slip boundary conditions rather than non-slip ones. It is more realistic to consider the impact of slip-on boundary layer flow due to a diminishing surface. Examining the slip influence over the stagnation point flows approaching a diminishing surface becomes crucial in various applications. Observing and identifying many solutions to boundary layer problems can be difficult in real-world situations. As a result, many researchers overlook a number of possibilities. Therefore, the effect of Brownian diffusion and thermophoresis on a point flow over an MHD boundary layer with a declining surface is examined in this work. For pragmatic reasons, partial slip is also taken into account. The shooting technique is used to solve the transforming equations numerically by similarity transformation. Finding and visualizing multiple boundary layer problem solutions might be difficult in real-world situations. As a result, many researchers overlook other options. Numerous MHD fluid flow solutions have been conceptually and quantitatively studied by numerous academics.

2. Related Work

The study of magnetohydrodynamic (MHD) nanofluid flow with partial slip has gained considerable attention due to its applications in industrial and biomedical processes. Numerous scholars have examined how chemical reactions and Soret and Dufour diffusivities affect the mass and heat transfer properties in such flows. Studies by [17], [18], [19], and [20] looked at the combined impacts of Soret and Dufour diffusion and emphasized how they enhance cross-diffusion phenomena. Research on chemical reactions, especially homogeneous and heterogeneous reactions, was conducted by [21], [22], [23], and [24] affect the stability and transport properties of nanofluid flows under magnetic fields. The importance of partial slip circumstances in boundary layer control was highlighted according the investigations of [25], [26], [27], and [28]. This role was frequently overlooked in previous models. Additionally, studies by [29], [30], [31], [32], and [33] developed numerical models accounting for combined thermal radiation, reactive species, and MHD effects, showing how multiple factors can influence entropy generation, velocity profiles, and temperature distribution. These studies collectively contribute to a deeper understanding of nanofluid behavior in advanced thermal systems under multi-physical influences. Other relevant researches include [34- 48].

3. Description of the Problem

The uninterrupted flow of a two-dimensional laminar boundary layer stagnation points down a linearly falling surface, is examined in this article. The x-axis is measured along the y-axis, which is perpendicular to the sheet. The flow is located at $y \geq 0$. For a viscous fluid flow that is incompressible and has partial slip it is expected that the decreasing sheet's velocity approximates the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_0}{\rho}(u - U_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho Cp}\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho Cp}\frac{\partial q_r}{\partial y} + D_{CT}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_{TC}\frac{\partial^2 C}{\partial y^2} - K_1(c - c_\infty) \tag{4}$$

Velocities in the direction of the x and y axes are denoted by the letters u and v, correspondingly. Boundary conditions, particularly partial slide, apply.

$$u = cx + L\left(\frac{\partial u}{\partial y}\right), v = 0, T = T_w, C = C_w at\ y = 0,$$

$$u \to U(x) = ax, T \to T_\infty, C \to C_\infty\ as\ y \to \infty \tag{5}$$

U(x) = ax represents the free stream velocity, while u = cx represents the stretched sheet velocity.

4. Method of solution

When [34] solve the mathematical model, they took into account the following dimensionless parameters, functions, and similarity transformations:

$$\eta = \sqrt{\left(\frac{a}{v}\right)}y, \psi = \sqrt{ax}xf(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty}, u = \frac{\partial\psi}{\partial y},$$

$$v = -\frac{\partial\psi}{\partial y}, \phi = \frac{C - C_\infty}{C_w - C_\infty}. \tag{6}$$

Using the aforementioned transformations into equations 7 through 10, the following system of equations are obtained:

$$f''' + ff'' - f'^2 + 1 - M(f' - 1) = 0, \tag{7}$$

$$(3Rd + 4)\theta'' + \Pr(f\theta' - f'\theta) + Ld\phi'\theta' = 0, \tag{8}$$

$$Nd\phi'' + Sc(f\phi' - f'\phi) - \lambda\phi = 0, \tag{9}$$

In this case, prime represents the differentiation with respect to η, whereas M stands for the magnetic parameter, Rd for the thermal radiation parameter, and Pr for the Prandtl number, Ld stands for the Soret diffusivity, Nd is the Dufour diffusivity, Sc, $\lambda, \varepsilon\ and\ \delta$ are the Schmidt number, Chemical reaction, velocity ratio parameter and velocity slip parameter. With these transformations, the continuity Eq. (1) satisfies in the same way. Its accompanying boundary conditions are reduced to:

$$f(0) = 0, f'(0) = \varepsilon + \delta f''(0), \theta(0) = 1, \phi(0) = 1, at\ \eta = 0$$

$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0, at\ \eta = \infty. \tag{10}$$

The set of equations (7), (8), and (9) is numerically solved using the shooting technique, with (10) displaying the boundary conditions for the Runge-Kutta method of order four, where c/a represents slipping parameter together with flow speed by $\delta = L(a/\upsilon)\,2$. Equations (1), (2) and (3) are transformed into an order one differential equation system as follows:

$$f_1{}' = f_2$$

$$f_2{}' = f_3$$

$$f_3{}' = f_2{}' - f_1 f_3 - 1 + M(1 - f_2)$$

$$f_4{}' = f_5$$

$$f_5{}' = -\frac{1}{(3Rd - 4)}\left[\Pr(f\theta' - f'\theta) + Ld\phi'\theta'\right]$$

$$f_6{}' = f_7$$

$$f_7{}' = -\frac{1}{Nd}\left[Sc(f\phi' - f'\phi)\right] - \lambda\phi \tag{11}$$

Where:

$$
\begin{aligned}
f &= f_1 \\
f' &= f_2 \\
f'' &= f_3 \\
f''' &= f_3{}' \\
\theta &= f_4 \\
\theta' &= f_5 \\
\theta'' &= f_5{}' \\
\phi &= f_6 \\
\phi' &= f_7 \\
\phi'' &= f_{7'}
\end{aligned}
\tag{12}
$$

$$
\left.
\begin{aligned}
&f_1(0) = 0,\ f_2(0) = \varepsilon + \delta f_3(0),\ f_4(0),\ f_6(0) = 1, \\
&f_2(\infty) = 1,\ f_4(\infty) = 0,\ f_6(\infty) = 0,
\end{aligned}
\right\}
\tag{13}
$$

Similar to the shooting procedure, initial guesses for and $\theta'(0)$ are regarded as s_1 and s_2, respectively, to be able to obtain the Initial Value Problem system solution, even though there were no these convictions provided in the referenced B.C. In order to obtain better and more accurate solutions, the calculated values were compared using the supplied boundary conditions and adjustment values according to the secant technique. A step size of h = 0.01 was used. Until accurate findings are acquired up to a 6–10 accuracy level, this process is repeated.

5. Results and Discussion

Equations (7-10) were numerically solved using Maple 14.0, taking boundary conditions into consideration. To answer numerical boundary value issues, this program defaults to using the fourth-fifth order Runge-Kutta-Felberg algorithm. By comparing our findings to previous research, we were able to establish their correctness and robustness.

These findings apply to a situation in which the fluid is a regular fluid without nanoparticles and the flat plate is kept at a constant temperature without the use of a mass suction, sink, or heat source.
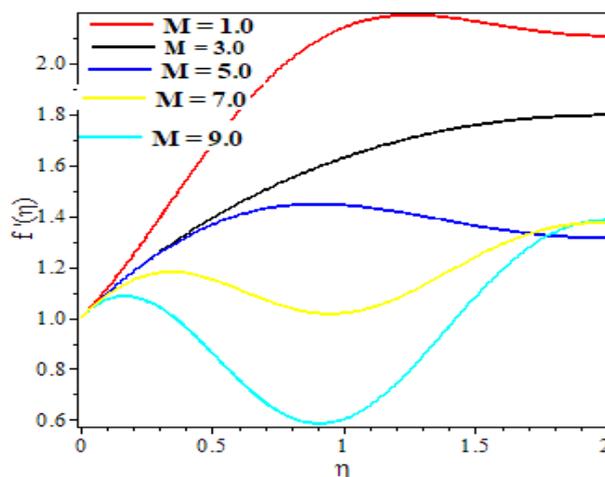
Figure 1: Influence of magnetic parameter on Momentum profile

The strength of the magnetic field's effect on a conducting fluid is commonly referred to as the magnetic parameter. This has intriguing effects on momentum. First, a Lorentz force is created when a magnetic field interacts with a moving conducting fluid, such as liquid metal or plasma. Similar to friction, this force creates a resistive effect by acting against the fluid's velocity. Stronger Lorentz forces, which slow down the fluid, are also a result of stronger magnetic fields. As a result, the momentum profile decreases and fluid particle velocity decreases, particularly in the vicinity of barriers. Additionally, by reducing velocity gradients and turbulent behavior, the magnetic field effectively damps the motion. In border layers, this effect is more apparent. Lastly, flatter velocity profiles result from less mass and energy being transported by fluid motion due to less momentum. Figure 1 makes it evident that any rise in the magnetic parameter results in a significant drop in the system's momentum.
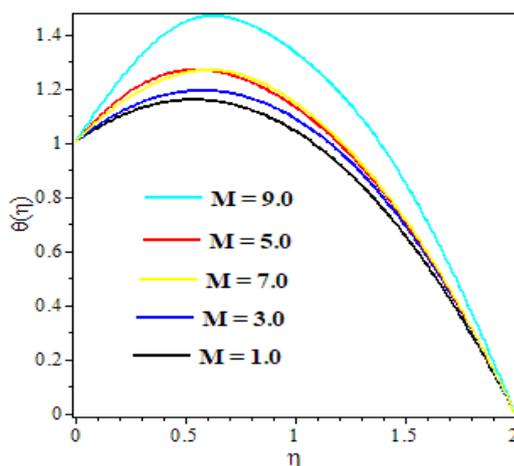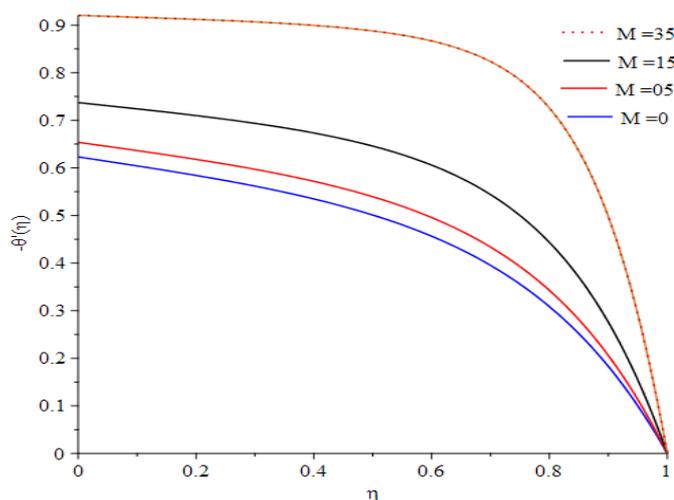

Figure 2: Influence of magnetic parameter on Temperature profile

The impact of a magnetic field on fluid flow is represented by the magnetic parameter (M). Increasing the magnetic parameter has the following effects: convection becomes less effective as fluid velocity decreases, making it more difficult for heat to escape the system. Also, because heat transfer slows, temperature within the fluid tends to rise leading to a steeper temperature profile. Similarly, lower flow usually correlates with less thermal dispersion, so the heat gets "trapped" more effectively in localized zones. Observing figure 2, it will be seen that the system's temperature rises as the magnetic parameter values are increased.

On the contrary, figure 3 is plotted to depict the influence of magnetic parameter on the nanoparticle concentration profile: it can be observed that for any increment in the parameter, the concentration profile is seen to decrease. This is

because when a magnetic field interacts with a conducting fluid, several things happen that can lead to a decrease in concentration profiles: firstly, magnetic field applies a Lorentz force on charged particles in the fluid. This force tends to oppose fluid motion, especially the flow velocity. With reduced fluid movement, the transportation of solute by convection slows down, diminishing concentration levels further away from the source. Also, the magnetic field introduces a viscous-like damping effect, often called magnetic damping. The stronger this effect (i.e., the larger the magnetic parameter), the slower the fluid moves. Concentration gradients diminish as a result of slowing down both velocity and mass diffusion. Moreover, the mass transfer rate of particles or chemical species decreases with decreasing flow velocity. The magnetic field essentially works as a "brake" on convection and diffusion processes. Additionally, the magnetic field can cause the boundary layer surrounding surfaces to thicken, which also slows diffusion and lowers the concentration profile across the domain. This behavior is predicated on laminar, steady-state MHD flow through channels or over surfaces. It is frequently investigated in engineering systems such as cooling electronic components, polymer extrusion, and nuclear reactors.


Figure 3: Influence of magnetic parameter on Nusselt number profile

The impact of the magnetic parameter on the Nusselt number profile, however, is displayed in Figure 3. Upon closer inspection, it is evident that the Nusselt number profile becomes more intense as the magnetic parameter values rise. The degree to which heat transport is improved over pure conduction is indicated by the Nusselt number. The Lorentz force can steady the flow and occasionally thicken the thermal boundary layer as the magnetic parameter rises. However, a higher magnetic field can improve temperature gradients close to the surface in some configurations (particularly with forced convection and particular boundary conditions), enhancing the rate of heat transfer. Convective heat transmission has increased as a result, as shown by a greater Nusselt number.
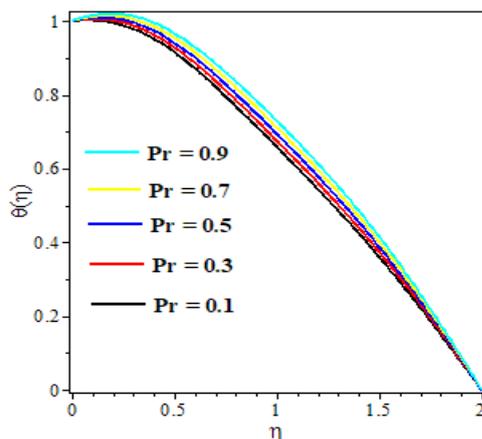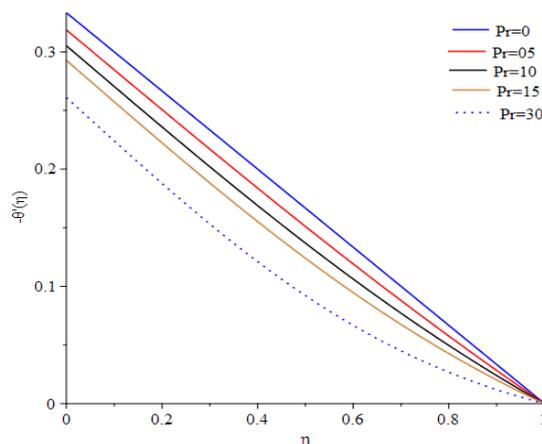

Figure 4: Influence of Prandtl number parameter on Temperature profile
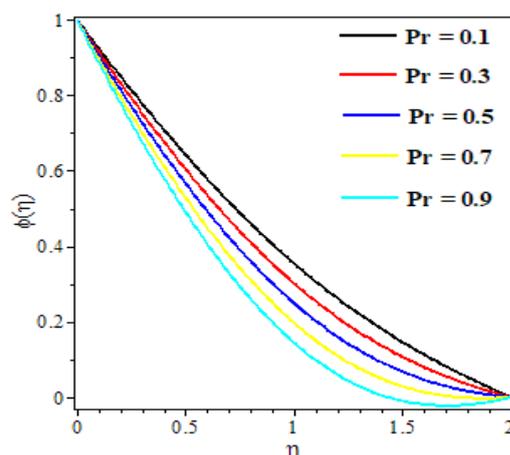
**Figure 5:** Influence of Prandtl number parameter on Concentration profile

The Prandtl number (Pr) plays a key role in shaping the temperature distribution in magnetohydrodynamic (MHD) fluid flows. This is how it operates: It calculates the momentum diffusivity to heat diffusivity ratio. A higher Prandtl number denotes poor thermal diffusivity, which causes heat to move through the fluid more slowly. This leads to a steeper temperature gradient close to the surface and a thinner thermal boundary layer, which causes the temperature to drop off from the heated surface more abruptly. Conversely, a lower Prandtl Number denotes high thermal diffusivity, which facilitates the transfer of heat and results in a wider temperature profile with slower variations.
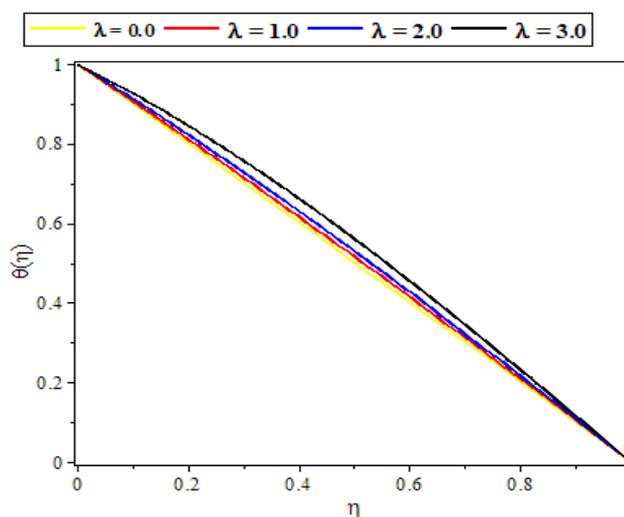

Figure 6: Influence of Prandtl number parameter on Nusselt number profile

Fluid velocity is dampened by Lorentz forces, which are introduced when a magnetic field is present. The magnetic field further reduces convective effects as the Prandtl number rises, confining thermal energy closer to the surface. As shown in fig. 5 above, this combination causes the overall temperature profile away from the surface to decline. Complexity is increased in several research by treating viscosity and Prandtl number as temperature-dependent. For instance, these parameters inversely change with temperature in natural convection flows over spheres, which further affects the behavior of heat transfer. Figure 6 shows similar effects: the nanoparticle concentration profile decreases with increasing Prandtl number parameter values. This is because higher values of Prandtl number have these effects: Temperature profile decreases due to reduced thermal diffusion, this weakens thermophoretic forces, which rely on temperature gradients to push nanoparticles. As a result, fewer nanoparticles migrate, leading to a lower concentration profile in the boundary layer. Additionally, Magnetic fields (MHD effects) and viscous dissipation also interact with Prandtl number dynamics, further modifying how nanoparticles distribute. Also, Higher Pr values can enhance fluid stability but reduce mixing, which also contributes to lower concentration. Reactive species concentrations are frequently lowered by increased chemical reaction rates. This may change the fluid's energy distribution, particularly if the species

is involved in heat generation or thermal conductivity. The amount of energy available to maintain or raise temperature can be decreased by chemical reactions that reroute energy away from thermal activities and toward molecular transformation. The fluid may heat as a result of dilution or changed thermal gradients when the rate of chemical reactions increases.
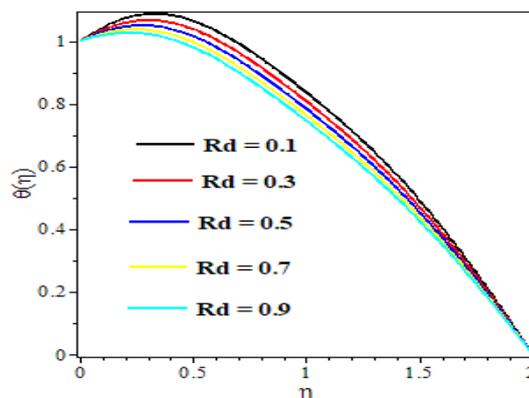
Figure 7: Influence of radiation parameter on temperature profile


**Figure 8:** Influence of chemical reaction parameter on temperature profile

Furthermore, figure 7 shows variation in the Nusselt number profile in regard to change of values in the Prandtl number. It can be observed that as the values of the parameter increases the Nusselt number profile is decreased.

Figure 8 is plotted to depict the influence of radiation parameter on the temperature profile: it can be observed that as the values of the parameter increases to lead to the grows in the temperature profile. This happens because, the radiation parameter plays a significant role in how thermal energy is exchanged. Here's a breakdown of why increasing the radiation parameter tends to increase the temperature profile: firstly, higher radiation parameter indicates stronger thermal radiation, this means more heat energy escapes from the fluid to its surroundings. As a result of that the fluid's internal temperature rises, increasing the temperature profile overall. Secondly, with more energy radiated away, the amount of heat retained by the fluid for conduction and convection decreases, also, the temperature gradient becomes high steep, especially near boundary layers. Thirdly, in MHD flows, magnetic fields resist motion and generate heating via Joule dissipation, but when radiative cooling dominates, it counteracts this magnetic heating effect, leading to net temperature enlarges. In addition to that, radiation can thick the thermal boundary layer, this thickening leads to faster heating of the fluid near surfaces, further contributing to a cooler overall profile. In engineering applications (e.g., cooling systems, astrophysical flows, plasma containment), controlling the radiation parameter helps modulate heat transfer efficiency. High radiation might be desirable when cooling is the goal.

Furthermore, figure 9, is designed to show the temperature variation in respect to the increase in values of chemical reaction parameter. It is evident that any increase in the parameter's values causes the temperature profile to rise. It occurs because, the chemical reaction parameter influences the interaction between species concentration and thermal energy. When this parameter increases, it typically signifies a stronger or more rapid chemical reaction occurring within the fluid. Many chemical reactions absorb heat from the surrounding fluid. As the reaction rate increases, more thermal energy is consumed, leading to a rise in temperature.
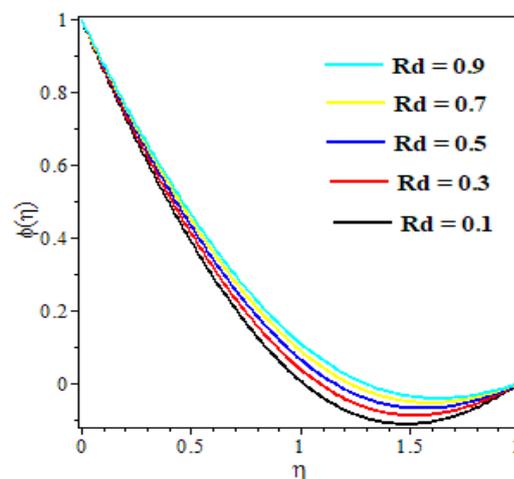
Figure 9: Influence of radiation parameter on Concentration profile
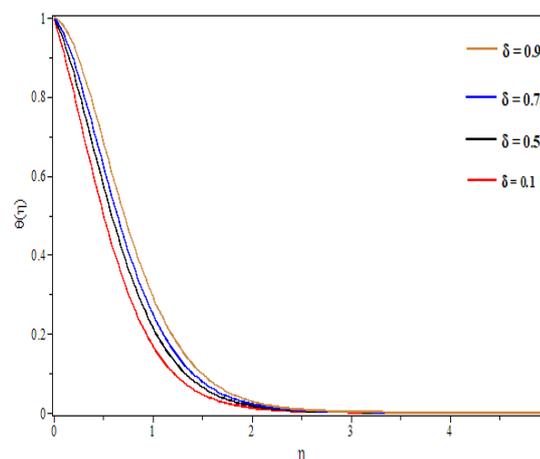
Figure 10: Influence of velocity slip parameter on temperature profile

On the other hand, the variation of the nanoparticle concentration profile with different radiation parameter values is depicted in Figure 9. It is evident that the profile decreases as the parameter's values rise. The concentration profile is significantly shaped by the radiation parameter, and raising it frequently results in a drop in concentration. The fluid's energy balance is changed when heat radiation is added to the system. Thermal radiation increases the temperature of the fluid near the surface. Brownian motion and thermophoresis, two important processes in nanofluid transport, are accelerated by this increased heat energy. Higher radiation, however, also speeds up diffusion, which makes nanoparticles disperse farther from the surface. The density of nanoparticle distribution throughout the flow domain is referred to as the concentration profile. A reduction indicates that there are fewer particles in the boundary layer or close to the surface. Higher thermal energy causes particles to disperse more widely, which lowers local concentration. By lowering the density gradient, radiative heat flow lessens the force that propels mass transport. Figure 10 shows how the velocity slip parameter affects the temperature profile. As the parameter's values grow, the temperature profile is observed to climb. The no-slip condition in classical fluid mechanics is predicated on the fluid velocity at the boundary

being equal to that of the boundary. The fluid can "slip" across the boundary due to velocity slip, hence the fluid velocity at the wall is not zero. In micro/nano-scale flows, whereby surface effects predominate, this is particularly pertinent. Nanoparticles can disperse more readily when there is less shear tension at the wall as a result of sliding. The total concentration profile rises as a result of the increased concentration of nanoparticles further from the wall. Furthermore, as the Dufour diffusivity value increases, the temperature profile rises. Figure 11 provides a detailed explanation of this. In magnetohydrodynamic (MHD) nanofluid flows, the Dufour effect—also referred to as the diffusion-thermo effect—plays a minor but significant role. It raises the temperature in the fluid domain by adding thermal energy to the system. The movement of particles in a fluid caused by temperature gradients is known as the Soret effect. This indicates that nanoparticles in nanofluids move from hotter to colder areas, affecting thermal behavior and concentration. Thermal energy is carried by nanoparticles when they move as a result of the Soret effect. Particularly close to the heated surface, this redistribution can lower the local temperature in areas where particles are departing.
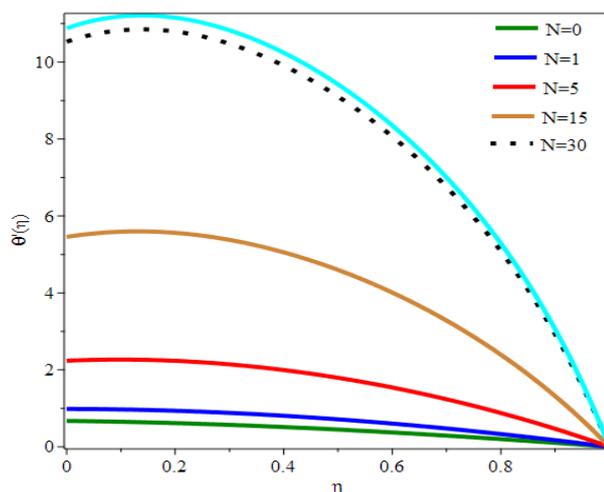

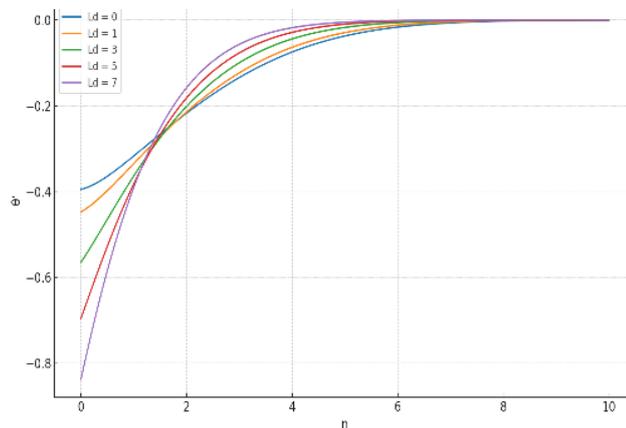**Figure 11:** Influence of Dufour diffusivity parameter on temperature profile

Figure 12: Influence of Soret diffusivity parameter on temperature profile

But when Soret dominates, the net result may be a temperature drop, especially in systems with weak Dufour influence. This is depicted on figure 12. Mass diffusion is enhanced more than the diffusion of thermal energy by the Soret effect. When combined with magnetic field effects that inhibit fluid mobility, this imbalance may result in a smaller temperature gradient. Fluid motion is resisted by the Lorentz force introduced by the magnetic field. This damping effect can lower the temperature profile by further reducing convective heat transfer when paired with Soret driven particle migration. Concentration gradient can cause the Dofour effect, which is the opposite of Soret, to raise temperature.

6. Conclusion

In this research, we embarked on a comprehensive investigation into the heat transfer within hybrid MHD nanofluid flow, incorporating the influence of Soret diffusivity, Dufour diffusivity, as well as chemical reaction. Mass diffusion is more enhanced by the Soret effects than inhibits fluid velocity, this imbalance may result in a reduce temperature gradient. Fluid motion is resisted by a Lorentz force introduced by the magnetic field. This damping effect can lower the temperature profile by further reducing convective heat transfer in conjunction with Soret- driven particle movement. Concentration gradient can raise the temperature by the Dofour effect, which is the opposite of Soret.

1. The momentum profile shows decrement when subjected to higher values of magnetic effect.
2. Furthermore, the temperature of the system is enhanced when affected by higher values of Prandtl number as well as Soret diffusivity.
3. However, in terms of temperature, the opposite is true with radiation parameter, Dufour diffusivity together with chemical reaction as well as chemical reaction parameter.
4. Additionally, nanoparticle concentration profile is observed to decrease when subjected to the higher values of Prandtl number as well as radiation parameter.
5. Finally, Nusselt number profile is a decreasing function with Prandtl number.

Conflicts of Interest

The author declares no conflicts of interest.

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